Key Control Strategies Emerge in Spring Loaded Inverted Pendulum Traversal of Slippery Terrain

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1 Introduction

Human locomotion is an intrinsically unstable and dynamic task. At any moment, an unexpected loss of ground reaction force (GRF) control due to slippery terrain can endanger this precarious equilibrium and necessitate a rapid reaction to maintain balance. Past studies into human slip recovery reflexes have discovered a region of the center of mass (COM) state space, called the feasible stability region, within which recovery from slippage is possible [1] [2]. Furthermore, research shows that human subjects engage in multiple different strategies for traversing low-friction patches, primarily characterized by a pronounced base of support (BOS) shift [3].

Conventional analyses of human slippage have made extensive use of the inverted pendulum model, while the spring loaded inverted pendulum (SLIP) has long been considered an accurate model of running animals and robots [4] [5]. Studies of SLIP dynamics have mostly disregarded ground slippage (i.e., infinite friction) and largely focus on limit cycle planning over multiple steps [6]. Only recently has some preliminary progress been made in understanding SLIP dynamics on low-friction terrain [7] [8]. In the high-friction case, one popular tool for analyzing SLIP limit cycles has been numerical trajectory optimization, which iteratively finds an objective-minimizing trajectory subject to various equality and inequality constraints [4] [6].

This study applies numerical trajectory optimization techniques to investigate traversal strategies for unforseen slippery patches. Unlike past studies of low-friction terrain that primarily used the inverted pendulum model with a substantial BOS [9], here we analyze a point-foot SLIP model common in dynamic locomotion literature. We also hope to discover any parallels between optimal SLIP solutions and human recovery strategies.

2 Optimization

Our SLIP incorporated a series damper, linear actuator (second derivative of neutral spring length), hip torque actuator, and small toe mass (10% of hip weight). Parameters were taken from [6] and are roughly equivalent to those found in human running. A toe mass was added to enable toe dynamics in the sliding stance phase.

Each simulated test scenario consisted of a randomly sized low-friction patch ($\mu = 0.05$) surrounded by sticky terrain. The SLIP was initialized contacting the patch with a plausible state for forwards walking (positive COM x velocity, negative COM y velocity, zero spring compression, toe in front of COM). Two possible step sequences were then considered: stepping immediately forward to the high-friction ground on the right (2 stance phases), or first stepping backward and then leaping over the patch (3 stance phases). Step sequences involving more than one slippery stance phase were not considered since in concrete applications low-friction surfaces complicate the control task and increase the risk of falling.

To calculate the minimum mechanical work required for each step sequence, the trajectory optimization problem was transcribed into a function optimization problem using direct collocation. Each stance phase was discretized into ten evenly-spaced nodes, with an additional parameter for the phase time duration. Dynamics between nodes were enforced using trapezoidal quadrature. To link the end of one stance phase with the beginning of the next, a flight time parameter was included, and the end COM state of the ballistic trajectory was constrained to equal the COM state at the start of the next phase. The two candidate step sequences were formulated as separate optimization problems. After transcription, the programming problem was solved using MATLAB's *fmincon* function and the Sequential Quadratic Programming algorithm.

To facilitate more reliable convergence, the optimization was performed in three steps. First, the optimizer was given a constant objective function—in other words, find any trajectory that satisfies the constraints. The output trajectory was then fed into an impulse-squared objective function, which is relatively fast and reliable to optimize. Finally, the output of the impulse objective was fed into a mechanical work objective function. This three-part method converged reliably from random initial guesses when solutions existed, usually in under two minutes.

3 Results and Discussion

The numerical trajectory optimizer was applied separately to both the 2-stance-phase and 3-stance-phase step sequences for several hundred randomly generated scenarios. Multiple distinctive control strategies emerged for the two cases (Figure 1).

Interestingly, some of these trajectories (especially in the 2-stance-phase case) validate findings from human experimentation. In [3], it was found that humans traversing an artificial low-friction patch used either a “walk-over” or “skate-over” strategy, where a walk-over meant that the BOS moved by less than 5cm during stance. These strategies were also discovered by the optimizer (Figure 1C and Figure 1D), showing that the SLIP is an accurate model for understanding human traversal of slippery surfaces. Past research has primarily utilized the SLIP model for legged locomotion over infinite friction surfaces and relied on the simpler inverted pendulum model for slippery terrain; this study applies the former approach to better understand low-friction recovery reflexes. Additionally, the similarities between human behavior and the optimized trajectories indicate that mechanical work optimization has a fundamental relationship with trajectory stability. This suggests that at some level humans implicitly prioritize minimum-work trajectories when recovering from an unexpected loss of ground friction.

Future lines of research include examining the feasibility of a strategy-based controller, which would use the initial COM state to select from a collection of traversal strategies upon detecting low-friction contact. Efforts to discover the feasible stability region prevalent in humans through numerical optimization of the SLIP would also be of interest. In the biological realm, we hypothesize from this study that humans would utilize either a “reverse-push” or a “back-skate” strategy for traversing slippery patches with three stance phases—however, this must be verified. Finally, further research could be conducted into whether numerical optimization of certain trajectory stability criteria yields the same motion strategies that were discovered with the minimum work objective.

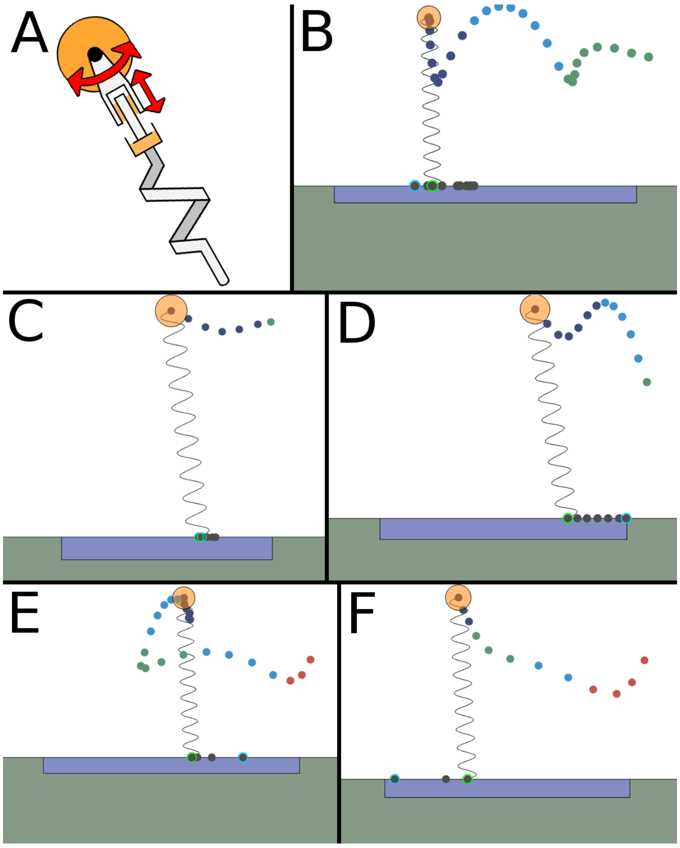

Figure 1: **A**: The Spring Loaded Inverted Pendulum (SLIP) model used in this experiment, with a spring, damper, and toe mass. The two red arrows represent actuation—one the hip torque, and the other the second derivative of the neutral spring length; **B**: Leap strategy; **C**: Walk-over strategy; **D**: Skate-over strategy; **E**: Reverse-push strategy; **F** Back-skate strategy.
Key: Dark blue (slide phase), green (first stick phase), red (second stick phase), light blue (flight phase), black (toe position), green highlight (landing), blue highlight (takeoff)—dots equally time-spaced, SLIP in starting state.

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